Predictions of the optimized Mechanical properties in Hydro fracturing process parameters using RSM Technique

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Abstract — In the hydrofraturing process, the parameters such as Pressure in N/mm^2, Temperature in ºC, Injection hole diameter in mm play a major role in determining the frature length during the hydrofracturing process. A central composite rotatable design with three factors and three levels was chosen to minimize the number of experimental conditions. An empirical relationship was established to predict the fracture length (mm) of the hydrofatruing process by incorporating independently controllable hydrofraturing process parameters. Response surface methodology (RSM) was applied to optimize the process parameters to attain maximum fracture length (mm).

Index Terms— Fracture length, Hydro mechanical, Hydro fracturing, Micro mechanical, Optimization, Response surface methodology

1 Introduction

The analysis of the hydro-mechanical behavior of rock masses remains an important topic in rock mechanics, due to it being a critical phenomenon in ongoing challenging issues such as tunneling under high groundwater pressures, extraction of hydrocarbons from deep, pressurized petroleum reservoirs, and underground nuclear waste disposal. Despite continuing and extensive efforts, such analysis continues to be difficult. Hydro-mechanical response in a rock mass is identified as the interaction between the solid phase of the rock materials and any interstitial fluid [1]. This technique involves pumping a fluid under pressure into a borehole. This pressurized fluid introduced into the borehole produces stress concentration in the surrounding rock causing the development of fractures due to micro mechanical effects [2]. Because of the heterogeneity of the material properties, rock structure and in situ stress state, the hydraulic fracturing process is highly complex [3].

A common difficulty in the hydraulic fracturing process in the real time is in observation and measurement of the fractures that develop beneath of the earth. Generally, the induced fracture geometry is measured by cutting the sample after the test [4],[5],[6] or by using an acoustic monitoring system [7],[8]. This method gives valuable results but limitations are there. The final results are observed by cutting the samples after the test. The resolution of the acoustic method is currently insufficient to capture details of the fracture propagation process. As a result, laboratory experiments on hydraulic fracturing in transparent materials have also been performed. These studies allowed the visualization in real time of the developing geometry of the fracture [9],[10] and the direction of fracture propagation [11],[12],[13]. Commonly used transparent geometrical analogues for fracturing are poly methyl methacrylate (PMMA, acrylic) [14],[15]. Since, the Fracture behavior is hard to predict because the relationship between stress and permeability is complex and highly dependent on pressure, temperature and Injection hole diameter.

The resulting fractures can be used to analysis the basis of hydraulic fracture propagation in real time field applications, the developed empirical relationship can be effectively used to predict the Fracture length in millimeters of Hydro fracturing process.

In this Research paper, It is well known that the input of hydrofraturing process parameters play a major role in determining the frature length. As the process facts have not been disclosed so far, the selection of input parameters to find the fracture length (mm) is very difficult. A common difficulty in the hydraulic fracturing process in the real time is in observation and measurement of the fractures that develop beneath of the earth. Hence, the problem of getting optimized hydrofraturing process parameters to attain maximum frature length is attempted in this investigation.

2 Experimental work

2.1 Fabricating The Experimental Set Up

A container for storing the fluid, a commercially available feed pump to feed pressurized fluid to the inner casing pipe provided in the PMMA test sample is shown in the experimental set up in fig 1. The PMMA test samples prepared for the experiment was 20 nos. The length of a

PMMA test sample was 300mm and its outside diameter was 150mm. The material in which the inner casing pipe made up of stainless steel and it has 6 to 10mm of an inner diameter. The applied pressure can be varied by the adjustment of the two control valves which is provided in the experimental set up and the range of pressure can be 4 to 8 N/mm^2. The adjustment of the flow control valves ensured the required pressure which was applied in the inner casing tube, before starting the experiment. A by pass line was provided separately in the experimental setup which helps to achieve the required pressure for the same. To control the pressurized fluid rate with respect to the time, say 5 sec to 15 mins, a 555 IC timer is provided for feed pump. For heating the PMMA test sample in the range of 40 to 60ºC, it is placed over the heater. The material in which the heater control unit made up of Nichrome and it has the capacity of 400W. The input to the heater was varied by the provision of the Dimmerstat 0-2A, Single phase, open type and the voltmeter and ammeter helps in the measurement of input. The digital range of voltmeter was 0 to 200V AC, the digital range of ammeter was 0 to 2A AC, the temperature indicator was digital 0 to 199.9ºC. AC single phase, 230V earthed stabilized current was the electrical supply for the experimental setup. The heat input at the desired value for the desired temperature on PMMA sample was adjusted by varying the Dimmerstat. The temperature was measured through the temperature gauge by the commercially available thermocouples that are embedded to the PMMA test sample. The experimental table and Stand was made up of MS square hollow pipe and angle. [16] From the literature, the predominant factors that have a greater influence on the Fracture rate of Hydro fracturing process had been identified. They were: (i) Pressure applied in N/mm^2 (ii) Temperature in ºC (iii) Injection hole diameter in mm. Large numbers of trial experiments were conducted to identify the feasible testing conditions for obtaining the Fracture length of Hydro fracturing process. The following inferences were obtained:

1. Based on the field trials the pressure applied is limited to 4 to 8 N/mm^2.
2. From the literature survey, the temperature and the injection hole diameter is limited to the range of 40 to 60 ºC and 6 to 10 mm respectively.
3. Further the Maximum with stand temperature of the PMMA samples is to be Less than 100ºC, hence the temperature range is fixed to 40 to 60 ºC only. [17]

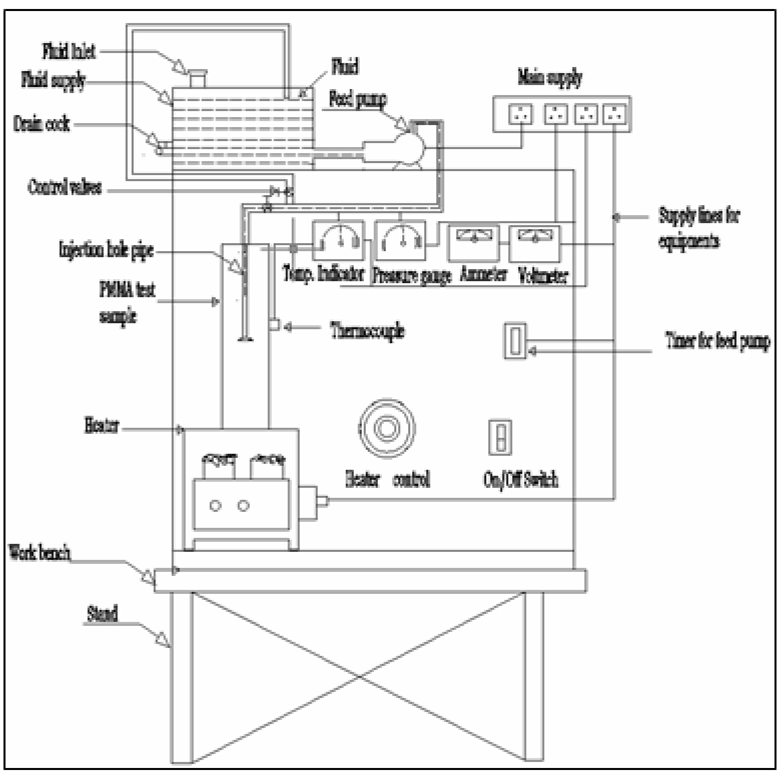

Fig.1 Experimental set up for Hydrofraturing process

2.2. Developing the experimental design matrix

Owing to a wide range of factors, the use of three factors and a central composite rotatable design matrix were chosen to minimize the number of experiments. A design matrix consisting of 20 sets of coded conditions (comprising a full replication three factorial of 8 points, six corner points and six center points) was chosen in this investigation. Table 1 represents the range of factors considered, and Table 2 shows the 20 sets of coded and actual values used to conduct the experiments. For the convenience of recording and processing experimental data, the upper and lower levels of the factors were coded here as +1.682 and -1.682 respectively. The coded values of any intermediate value could be calculated using the following relationship.

$$X_i=1.682[(2X-(X_{max}-X_{min})] / [X_{max}-X_{min}] \quad (1)$$

Where Xi is the required coded value of a variable X and X is any value of the variable from Xmin to Xmax, Xmin is the lower level of the variable; Xmax is the upper level of the variable.

TABLE 1

IMPORTANT FACTORS AND THEIR LEVELS

S. No	Factor	Unit	Notation	Levels				
				-1.682	-1	0	+1	+1.682
1	Pressure applied	N/mm^2	A	4.0	5.0	6.0	7.0	8.0
2	Temperature	ºC	B	40.0	45.0	50.0	55.0	60.0
3	Injection hole Diameter	mm	C	6.0	7.0	8.0	9.0	10.0

TABLE 2

DESIGN MATRIX AND EXPERIMENTAL RESULTS

Ex. No	Coded values			Actual Values			Fracture length (mm)
	Pressure applied (A)	Temperature (B)	Injection hole diameter (C)	Pressure applied (A)	Temperature (B)	Injection hole diameter (C)	
1	-1	-1	-1	5.00	45.00	7.00	210
2	+1	-1	-1	7.00	45.00	7.00	250
3	-1	+1	-1	5.00	55.00	7.00	200
4	+1	+1	-1	7.00	55.00	7.00	400
5	-1	-1	+1	5.00	45.00	9.00	240
6	+1	-1	+1	7.00	45.00	9.00	350
7	-1	+1	+1	5.00	55.00	9.00	360
8	+1	+1	+1	7.00	55.00	9.00	580
9	-1.682	0	0	4.32	50.00	8.00	220
10	+1.682	0	0	7.68	50.00	8.00	460
11	0	-1.682	0	6.00	41.59	8.00	210
12	0	+1.682	0	6.00	58.41	8.00	410
13	0	0	-1.682	6.00	50.00	6.32	260
14	0	0	+1.682	6.00	50.00	9.68	420
15	0	0	0	6.00	50.00	8.00	390
16	0	0	0	6.00	50.00	8.00	420
17	0	0	0	6.00	50.00	8.00	420
18	0	0	0	6.00	50.00	8.00	350
19	0	0	0	6.00	50.00	8.00	430
20	0	0	0	6.00	50.00	8.00	330

3 DEVELOPING AN EMPIRICAL RELATIONSHIP

In the present investigation, to correlate experimental test parameters and the Fracture length in Hydrofraturing process, a second order quadratic model was developed. The response (Fracture length) is a function of pressure applied in N/mm^2 (A), Temperature in ºC (B) and Injection hole diameter in mm (C) and it could be expressed as,

Fracture length (FL) = f {A,B,C} (2)

The empirical relationship must include the main and interaction effects of all factors and hence the selected polynomial is expressed as follows:

$$Y = b0 + \sum bi\ xi + \sum bii\ xi2 + \sum bij\ xi\ xj \quad (3)$$

For three factors, the selected polynomial could be expressed as

$$\text{Fracture length (FL)} = b_0 + b_1(A) + b_2(B) + b_3(C)+b_{11}(A^2)+b_{22}(B^2)+b_{33}(C^2)+b_{12}(AB)+b_{13}(AC)+ b_{23}(BC) \quad (4)$$

Where b_0 is the average of responses (Fracture length) and b_1, b_2, b_3, . . . b_{11}, b_{12}, b_{13}, . . . b_{22}, b_{23}, b_{33}, are the coefficients that depend on their respective main and interaction factors, which were calculated using the expression given below:

$$B_i = \sum(X_i, Y_i)/n \quad (5)$$

Where 'i' varies from 1 to n, in which X_i is the corresponding coded value of a factor and Y_i is the corresponding response output value (Fracture length) obtained from the experiment and 'n' is the total number of combination considered. All the coefficients were obtained applying central composite face centered design using the Design Expert statistical software package (Trial version 8.0.1). The significance of each coefficient was determined by Student's t test and p values, which are listed in Table 3. The Values of "Prob>F" less than 0.0500 indicate that model terms are significant. In this case, A, B, C, AB, BC, A^2, B^2 and C^2 are significant model terms.

The values greater than 0.10 indicate that the model terms are not significant. The results of multiple linear regression coefficients for the second-order response surface model are given in Table 3. The final empirical relationship was constructed using only these coefficients, and the developed final empirical relationship is given below:

Fracture Length =

$$\{+3.90+0.71*A+0.61*B+0.54*C+0.34*A*B+0.26*B*C-0.18*A^2-0.29*B^2-0.18*C^2\} \text{ mm} \quad (6)$$

The Analysis of Variance (ANOVA) technique was used to find the significant main and interaction factors. The results of second order response surface model fitting in the form of Analysis of Variance (ANOVA) are given in Table 4. The determination coefficient (R^2) indicated the goodness of fit for the model.

The Model F-value of 22.95 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. The values of "Prob > F" less than 0.0500 demonstrates a very high significance for the regression model. In this case A, B, C, AB, BC, A^2, B^2 and C^2 are significant model terms. The values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model. The goodness of fit of the model was checked by the determination coefficient (R^2).

The coefficient of determination (R^2) was calculated to be 0.9538 for response. This implies that 95.38% of experimental data confirms the compatibility with the data predicted by the model, and the model does not explain only 4.62% of the total variations. The R^2 value is always between 0 and 1, and its value indicates aptness of the model. For a good statistical model, R^2 value should be close to 1.0. The adjusted R^2 value reconstructs the expression with the significant terms. The value of the adjusted determination coefficient (Adj R^2=0.9123) is also high to advocate for a high significance of the model. The Pred. R^2 is 0.9081 that implies that the model could explain 90% of the variability in predicting new observations. This is in reasonable agreement with the Adj R^2 of 0.9123.

The value of coefficient of variation is also low as 8.85% indicate that the deviations between experimental and predicted values are low. Adeq precision measures the signal to noise ratio. A ratio greater than 4 is desirable. In this investigation, the ratio is 17.344, which indicates an adequate signal. This model can be used to navigate the design space. The normal probability of the Fracture length shown in Fig2 reveals the residuals were falling on the straight line, which meant that the errors were distributed normally. All of this indicated an excellent suitability of the regression model. Each of the observed values compared with the experimental values shown in Fig3.

TABLE 3

ESTIMATED REGRESSION COEFFICIENTS

Factor	Estimated co efficient
Intercept	3.90
A-Pressure	0.71
B-Temperature	0.61
C-Injection hole diameter	0.54
AB	0.34
AC	0.11
BC	0.26
A^2	-0.18
B^2	-0.29
C^2	-0.18

TABLE 4

ANOVA TEST RESULTS

Source	Sum of squares	df	Mean square	F Value	p-value prob.>F	
Model	19.33	9	2.15	22.95	< 0.0001	significant
A-Pressure	6.94	1	6.94	74.16	< 0.0001	
B-Temperature	5.00	1	5.00	53.43	< 0.0001	
C-Injection hole diameter	4.00	1	4.00	42.74	< 0.0001	
AB	0.91	1	0.91	9.74	0.0109	
AC	0.10	1	0.10	1.08	0.3228	
BC	0.55	1	0.55	5.89	0.0356	
A^2	0.48	1	0.48	5.12	0.0472	
B^2	1.20	1	1.20	12.80	0.0050	
C^2	0.48	1	0.48	5.12	0.0472	
Residual	0.94	10	0.094			
Lack of Fit	0.076	5	0.015	0.088	0.9907	not significant
Pure Error	0.86	5	0.17			
Cor.Total	20.27	19				

Std. Dev.	0.31	R-Squared	0.9538
Mean	3.46	Adj R-Squared	0.9123
C.V. %	8.85	Pred R-Squared	0.9081
PRESS	1.86	Adeq Precision	17.344

df -degrees of freedom, CV- coefficient of variation, F- Fisher's ratio, p- probability

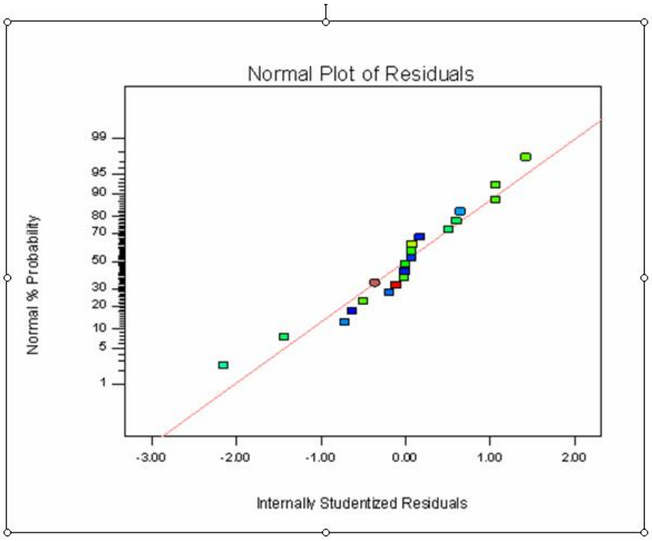

Fig. 2. Normal probability plot.

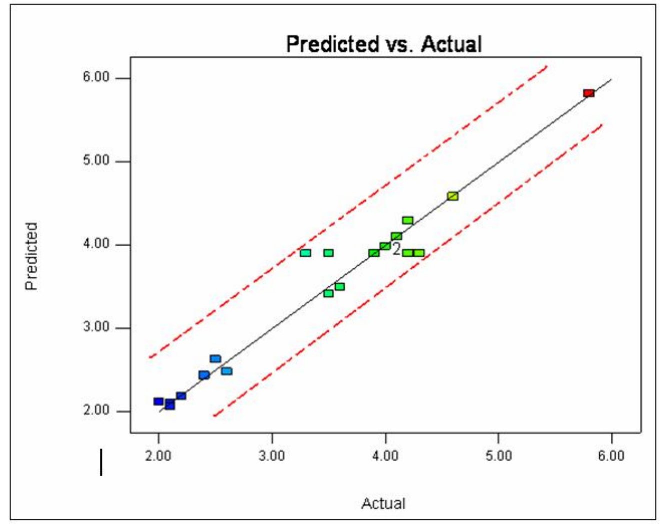

Fig. 3. Correlation graph for response (Fracture length)

4 OPTIMIZING THE HYDROFRATURING PROCESS PARAMETERS

The response surface methodology (RSM) was used to optimize the parameters in this study. RSM is a collection of mathematical and statistical techniques that are useful for designing a set of experiments, developing a mathematical model, analyzing for the optimum combination of input parameters, and expressing the values graphically [18]. To obtain the influencing nature and optimized condition of the process on Hydrofraturing, the surface plots and contour plots which are the indications of possible independence of factors have been developed for the proposed empirical relation by considering two parameters in the middle level and two parameters in the x and y axes as shown in Fig6. These response contours can help in the prediction of the response for any zone of the experimental domain [19].

The apex of the response plot shows the maximum achievable Fracture length (mm). A contour plot is produced to display the region of the optimal factor settings visually. For second-order responses, such a plot can be more complex compared to the simple series of parallel lines that can occur with first-order models. Once the stationary point is found, it is usually necessary to characterize the response surface in the immediate vicinity of the point. Characterization involves identifying whether the stationary point is a minimum response or maximum response or a saddle point. To classify this, it is most straightforward to examine it through a contour plot. Contour plots play a very important role in the study of a response surface. It is clear from Fig6 that the Fracture length increases with the increase of applied pressure (N/mm^2), Temperature (ºC) and Injection hole diameter (mm). By analyzing the response surfaces and contour plots in Fig5, the maximum achievable fracture length (mm) value is found to be 580mm. The corresponding parameters that yielded this maximum value are Temperature 55ºC and Injection hole diameter 9 mm. Contributions made by the process parameters on fracture length (mm) can be ranked [20] from their respective F ratio value which was seen in Table 3, provided the degrees of freedom are same for all the input parameters.

The higher F ratio value implies that the respective term is more significant and vice versa. From the F ratio values, it can be concluded that pressure (N/mm^2) is contributing more on fracture length (mm), and it is followed by temperature (ºC) and injection hole diameter (mm) for the range considered in this investigation. A maximum Fracture length (mm) of 580 mm obtained under the maximum value of applied pressure 7 N/mm^2, Temperature 55ºC and Injection hole diameter 9 mm during the experimental work shown in Fig4.

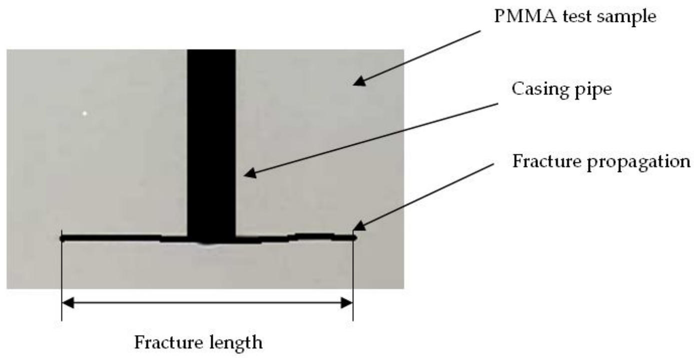

Fig. 4 Propagation of Fracture

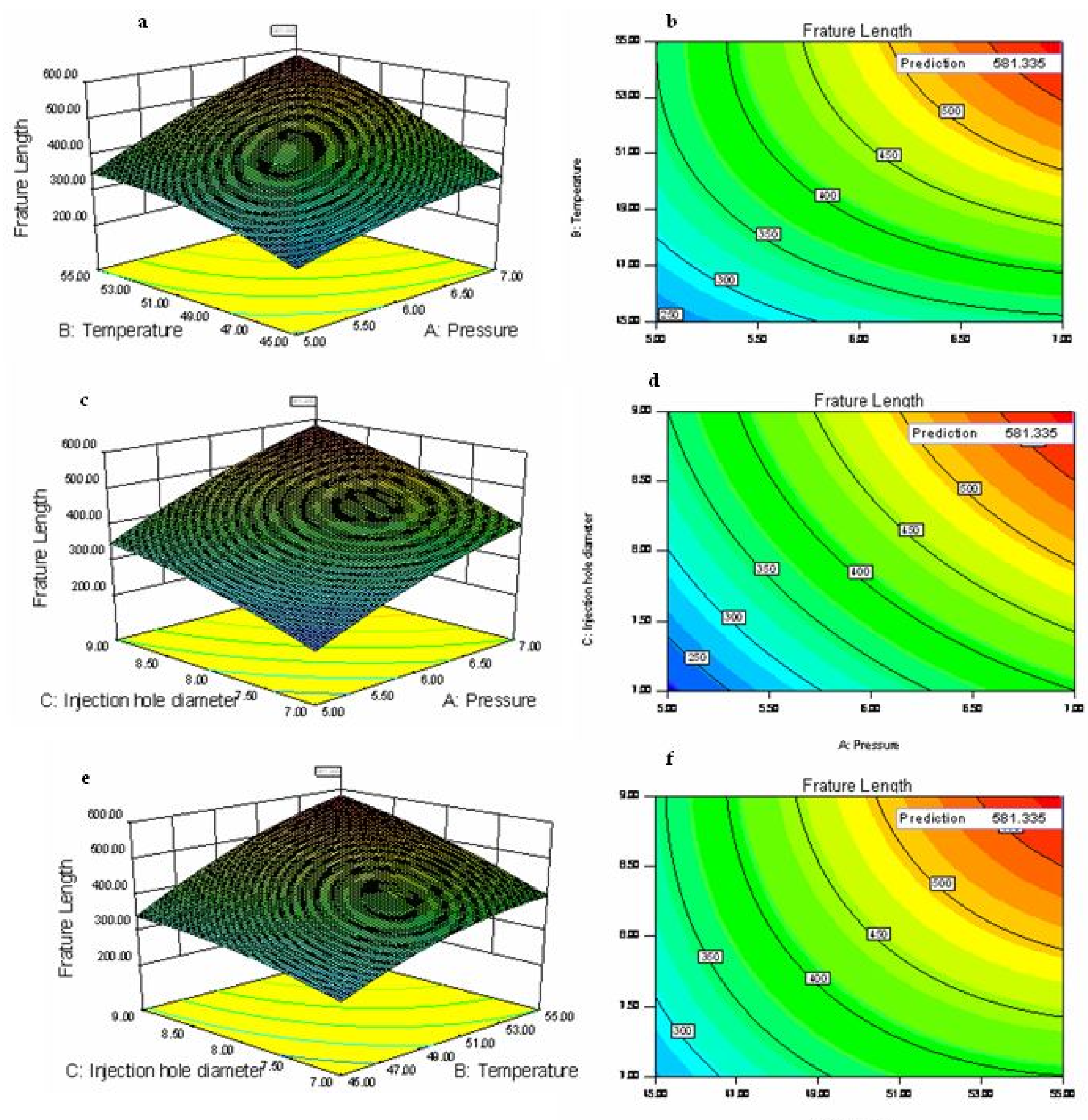

Fig. 5 Response graphs and contour plots. a, b Interaction effect of Pressure and Temperature. c, d Interaction effect of Pressure and Injection hole diameter e, f Interaction effect of Temperature and Injection hole.

5 CONCLUSIONS

1. An empirical relationship was developed to predict the Frature length (mm) as a mechanical property in hydrofraturing process incorporating parameters at 95% confidence level.

2. A maximum Fracture length (mm) of 581.335 mm could be attained under the maximum value of applied pressure (N/mm^2) Temperature and Injection hole diameter.

3. Of the three process parameters investigated, the applied pressure (N/mm^2) found to have greater influence on Fracture length (mm) followed by Temperature (ºC) and Injection hole diameter (mm).

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REFERENCES

[1] Rutqvist J, Stephansson .O," The role of hydro mechanical coupling in fractured rock engineering" Hydrogeol Journal 2003,11: 7–40.

[2] B.Guruprasad, A. Ragupathy, T.S. Badrinarayanan, K.B. Rajkumar, "The Stress Impact On Mechanical Properties Of Rocks In Hydro Fracturing Technique", International Journal of Engineering Science and Technology (IJEST), ISSN : 0975-5462 Vol. 4 No.02 February 2012, Pages 571-580.

[3] Germanovich, L. N., D. K. Astakhov, M. J. Mayerhofer, J. Shlyapobersky, and L. M. Ring , "Hydraulic fracture with multiple segments – I: Observations and model formulation", Int. J. Rock Mech. Min. Sci., 34, 471,1997.

[4] Murdoch, L. C, "Hydraulic fracturing of soil during laboratory experiments: part I Methods and observations", Geotechnique, 43, 255– 265,1993.

[5] de Pater, C. J., L. Weijers, M. Savic, K. H. A. A. Wolf, P. J. van den Hoek, and D. T. Barr , "Experimental study of nonlinear effects in hydraulic fracture propagation ", SPE Prod. Facil., 9, 239– 246,1994.

[6] Abass, H. H., S. Hedayati, and D. L. Meadows , "Non planar fracture propagation from a horizontal wellbore: Experimental study", SPE Prod. Facil., 11, 133– 137,1996.

[7] de Pater, C. J., L. Weijers, M. Savic, K. H. A. A. Wolf, P. J. van den Hoek, and D. T. Barr , "Experimental study of nonlinear effects in hydraulic fracture propagation ", SPE Prod. Facil., 9, 239– 246,1994

[8] Groenenboom, J., and D. B. van Dam, "Monitoring hydraulic fracture growth: Laboratory experiments", Geophysics, 65, 603–611.

[9] Rummel, F. (1987), "Fracture mechanics approach to hydraulic fracturing stress measurements in Fracture Mechanics of Rock", edited by B. K. Atkinson, pp. 217– 239, Elsevier, New York,2000

[10] Bunger, A. P., R. G. Jeffrey, and E. Detournay, "Toughness dominated near-surface hydraulic fracture experiments", paper presented at Gulf Rocks 2004, 6th NARMS: Rock Mechanics Across Borders and Disciplines, Am. Rock Mech. Assoc., Houston, Tex., 5 – 9 June,2004.

[11] Hubbert, M. K., and D. G.Willis, "Mechanics of hydraulic fracturing", J. Petrol. Technol., 9, 153– 166,1957.

[12] Takada, A., "Experimental study on propagation of liquid-filled crack in gelatin: Shape and velocity in hydrostatic stress condition", J. Geophys. Res., 95, 8471– 8481,1990.

[13] Bakala, M., "Fracture propagation in sediment-like materials", M.S. thesis, 83 pp., Univ. of Okla., Norman,1997.

[14] Rummel, F, "Fracture mechanics approach to hydraulic fracturing stress measurements, in Fracture Mechanics of Rock", edited by B. K. Atkinson, pp. 217– 239, Elsevier, New York,1987.

[15] Cooke, M. L., and D. D. Pollard, "Fracture propagation paths under mixed mode loading within rectangular blocks of Poly methyl methacrylate", J. Geophys. Res., 101, 3387– 3400, 1996.

[16] B. Guruprasad , A. Ragupathy, T.S. Badrinarayanan, R.Venkatesan "Estimation of Fracture Length as a Mechanical Property in Hydrofraturing Technique using an Experimental Setup" International Journal of Engineering and Technology (IJET-UK), ISSN 2049- 3444, Volume 2, No. 12, , Pages 1921-1925 , December, 2012.

[17] B. Guruprasad , A. Ragupathy, T.S. Badrinarayanan, R.Venkatesan "Estimation of Fracture Length as a Mechanical Property in Hydrofraturing Technique using an Experimental Setup" International Journal of Engineering and Technology (IJET-UK), ISSN 2049- 3444, Volume 2, No. 12, Pages 1921-1925, December, 2012.

[18]. Khuri AI, Cornell J "Response surfaces; design and analysis", Marcel Dekker, New York,1996.

[19] Tien CL, Lin SW"Optimization of process parameters of titanium dioxide films by response surfaces methodology" Opt Commun 266:574–581,2006.

[20] Phillip JR "Taguchi techniques for quality engineering", Mc Graw-Hill, New York,1988.

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